Stresses in bracings due to lateral torsional buckling of beams

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ABSTRACT: In structural engineering slender beams with I-sections are often supported by bracings. For the determination of the brace stresses due to the support, geometric imperfections can be applied to the supported beam in order to obtain an equivalent load. These imperfections are given in EC 3 for example.

This approach is appropriate for beams with only positive or only negative bending moments. For structures with negative bending moments at the beam ends and positive bending moments in midspan, as for example for the beam of a framework, the procedure using the equivalent imperfections does not provide realistic results.

The load bearing behaviour for beams with negative bending moments at the ends will be presented in this essay. Details on how equivalent imperfections should be applied and on the determination of brace stresses will be pointed out. Furthermore the influence of the rotational stiffness of adjacent members on the beam stability and the brace stresses will be investigated.

1 INTRODUCTION

Two determination models to obtain the brace stresses due to the support of the slender beams will be presented in this essay, see in Figure 1. These models are often used for the design in practise. The first model is from EC 3 and the second is an analysis using second order theory with equivalent geometric imperfections. Afterwards it will be shown with an example, how to determine the stresses in braces using second order theory.

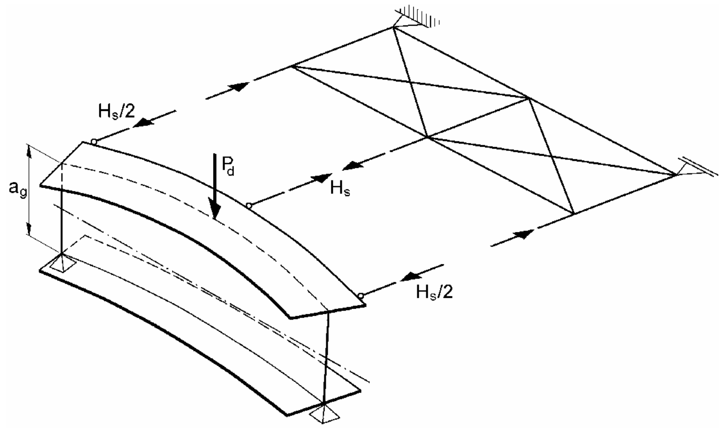


Figure 1 Beam with positive and negative bending moments supported by bracings

2 METHOD WITH AN COMPRESSION MEMBER ACCORDING TO EC 3

Basis of this model is a compression member, as shown in Figure 2. The member is deflected at midspan with an equivalent geometric imperfection $e_0 = \alpha_m L / 500$. The factor α_m regards the low probability, that all beams have the imperfection in the same direction. It depends on the number m of beams to be stabilized. The formula to determine α_m is given in Figure 2. δ_p is a factor to increase the brace stresses because of second order theory.

The equivalent geometric imperfection can also be converted into an equivalent load group. In that case the compression member is additionally loaded by a continuous load q and at the supports are reverse forces to fulfill the equilibrium. The formula for the continuous load q is also shown in Figure 2. The compression force N_{ed} in the flanges of the stabilized beams can be determined using the following formula:

$$N_{Ed} = \frac{M_y}{a_g} + \frac{N}{2} \qquad (1)$$

The internal forces of the posts, bracings and bracing chords can be calculated according to first order theory by the equivalent load group. If it is necessary to regard in second order theory, the additional abortive effects have to be considered.

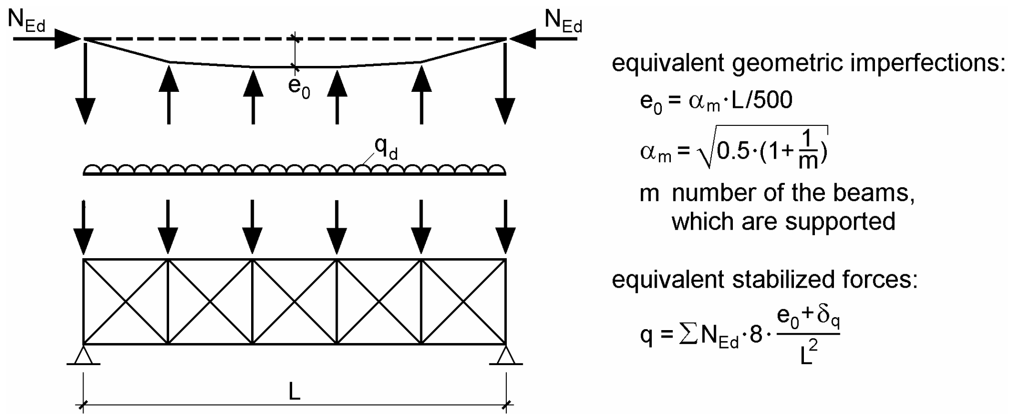


Figure 2 Determination model with a compression member from EC 3

Annotation: The origin of other calculation models, as for example of Petersen (1982) or Gerold (1963), is also the compression post. If constant or nearly constant pressure forces occur, these methods can be used for the verification of the bracings. Krahwinkel (2001) investigated a different method and the suitability for the verification of the bracings.

3 VERIFICATION WITH A SECOND ORDER THEORY ANALYSIS

Besides the methods described above it is possible to determine the stabilization stresses due to the lateral torsional buckling of beams with an analysis using second order theory. For such a determination you need the equivalent geometric imperfections. They are given in EC 3.

This analysis has the following advantages: The critical load factor, the internal forces of the beam and the stabilization stresses due to lateral torsional buckling can determined in one calculation. It is also possible to verify the load carrying capacity in the same step. The Finite Element Program KSTAB is used for the following analysis in this essay. The used program uses beam elements with seven degrees of freedom at each node. The theoretical background for these elements is described by Kindmann & Kraus (2007). The program is able to determine the internal forces according to second order theory with regard to the equivalent geometric imperfections. It is also possible to regard other stabilization arrangements like continuous springs, single springs, shear fields and torsional spring bedding acting at any point of the cross section.

4.1 *Description of the example*

A typical example of a warehouse, see Figure 3, is used to show the load carrying behaviour of the framework beam and to determine the stresses in the bracings due to the lateral torsional buckling of the beam.

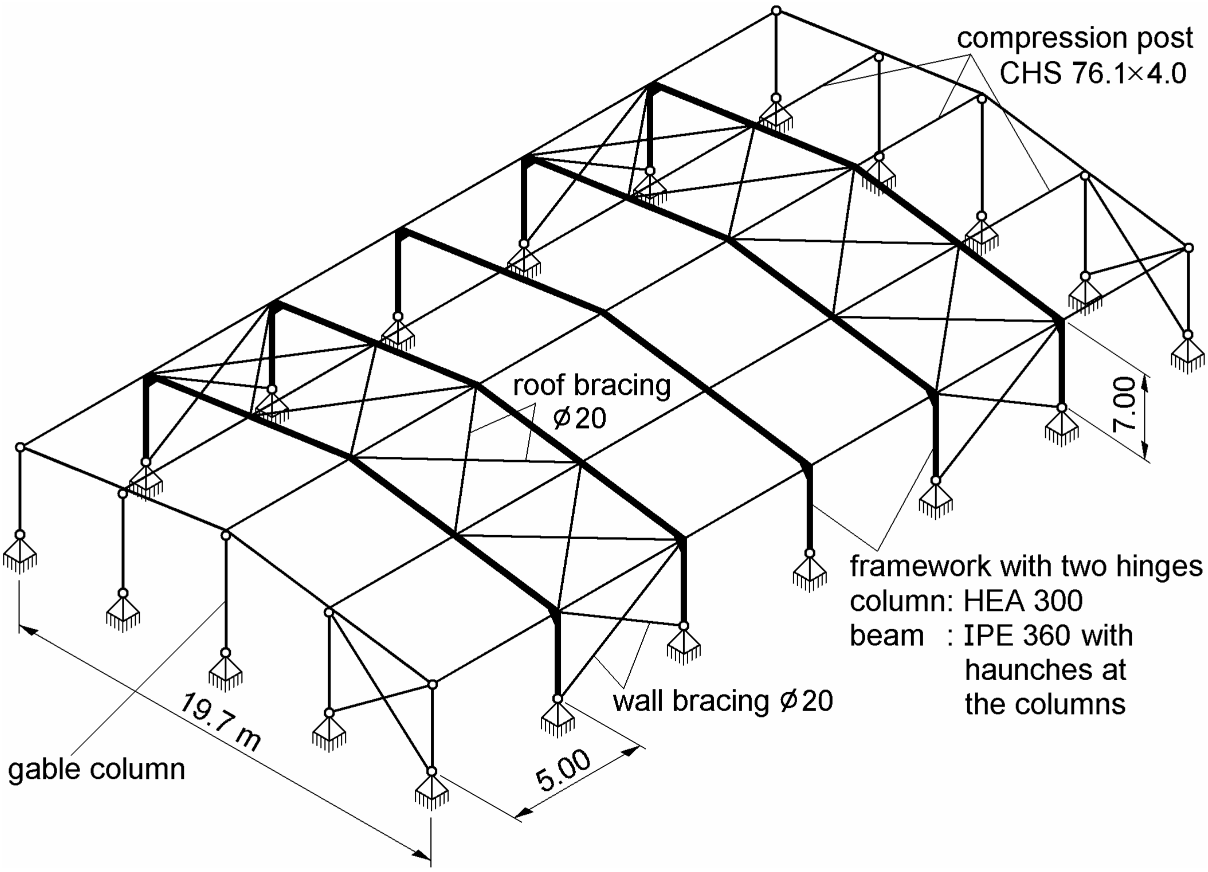
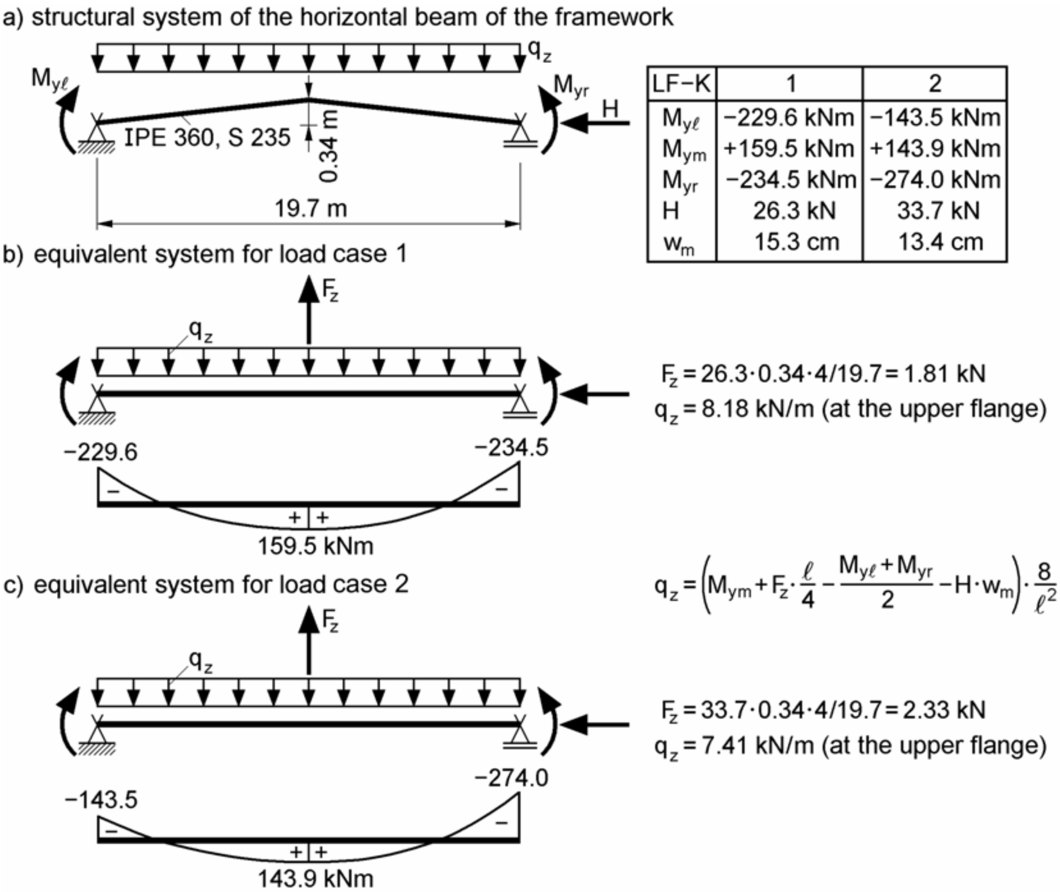


Figure 3 Isometric illustration of the example

The warehouse has five frameworks and two gable walls. More information about the determination of the internal forces of the framework are shown by Kindmann (2008). The plane structural system of the extracted beam with the acting loads is shown in Figure 4. They are loaded in vertical direction by the roof dead weight, snow loads and wind pressure on the roof.

The beams of the framework are often slender I-profils. That means, if these are loaded by compression and bending forces, they are vulnerable for the stability cases lateral buckling and lateral torsional buckling. The beams have to be stabilized by lateral supports, like shear fields due to the roof topping or bracings in the roof. Another component for the stabilization is a torsional spring bedding. The torsional spring bedding is a continuous torsional spring preventing the torsional rotation ϑ of the beam. The influence of the torsional spring bedding is shown for the stability of the beam with this example.

In this example only wind forces on the wall of the warehouse have to be regarded in horizontal direction. Two bracings in the roof and two bracings in each sidewall have to be able to carry the horizontal loads in longitudinal direction of the warehouse. The horizontal loads for the roof bracings are the wind loads at the walls and the stabilizing forces due to lateral buckling of the framework beam.



LF−K	1	2
$M_{y\ell}$	−229.6 kNm	−143.5 kNm
M_{ym}	+159.5 kNm	+143.9 kNm
M_{yr}	−234.5 kNm	−274.0 kNm
H	26.3 kN	33.7 kN
w_m	15.3 cm	13.4 cm

Figure 4 Decisive load cases for the framework beam

4.2 *Stabilization and lateral torsional buckling of the framework beam*

The static system of the framework beam is shown in Figure 4 for the following investigations. The loads are also shown in this figure. The next verifications are done for the load cases 1 and 2, see Figure 4. Because of the low roof pitch you can assume that the beam of the framework is nearly straight. To have the same distribution of the bending moment $M_y(x)$ a load has to be applied upwards at midspan. It arises from the condition $F_z \cdot \frac{\ell}{4} = H \cdot 0.34\ m$.

With regard to this example it is not possible to obtain a stable equilibrium without stabilizations. Without any stabilization the eigenvalue is $\eta_{Ki,d} = 0.220$ for load case 1 and $\eta_{Ki,d} = 0.237$ for load case 2. The beam can be stabilized by continuous torsional spring due to the roof topping and the horizontal springs due to the roof bracings. However, it is not sufficient to regard only bracings to stabilize highly utilized systems as shown in this example, since parts of the upper flange are compressed as well as the lower flange due to the positive and negative bending moments.

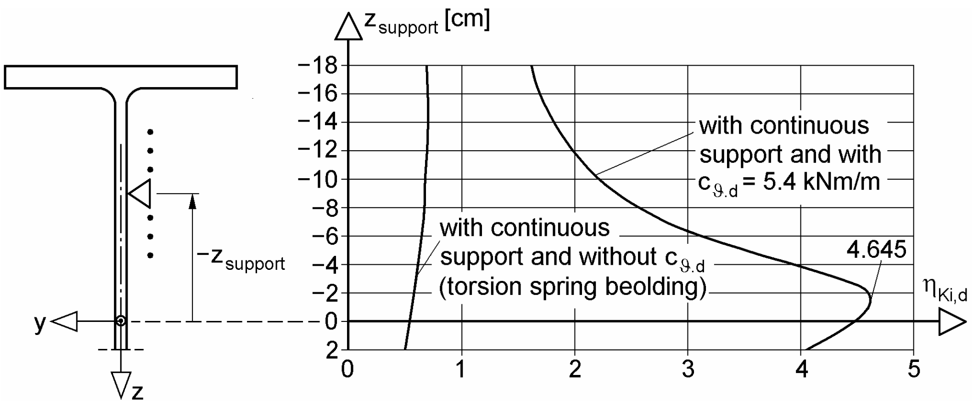


Figure 5 Critical load factor $\eta_{Ki,d}$ depending on $z_{support}$ for load case 2

If the continuous torsional spring is $c_\vartheta = 0$, the critical load factor $\eta_{Ki,d}$ is always less than one, see Figure 5. So it is necessary to use other stabilizations to obtain a stable equilibrium. The second curve is with a continuous torsional spring $c_\vartheta = 5.4$ kNm/m regarding the roof topping. The shown critical load factor depends on the acting point of the continuous horizontal support. The maximum critical load factor is $\eta_{Ki,d} = 4.645$ for a the horizontal support acting at $z_{support} = -1$ cm and a continuous torsional spring $c_\vartheta = 5.4$ kNm/m in this example.

Annotation: A continuous horizontal support is not available here, because the roof bracing is only connected at midspan and at the quarter points of the beam. One roof bracing stabilizes half of the roof and for the verifications it is assumed that the wind pressure on the gable wall will be transfered by the roof bracings. There are torsional internal forces in the beam because of the excentric connection at $z_{VB} = -9$ cm. They are low in this case and are therefore neglected.

For the next calculation of the framework beam the only support is the continuous torsional spring with a value of $c_{\vartheta,d} \cong 5.4\ \mathrm{kNm/m}$. The critical load factor is $\eta_{Ki,d} = 0.662$ for load case 1 and $\eta_{Ki,d} = 0.675$ for load case 2. The support by the continuous torsional spring is not sufficient as well. For that reason the support of the roof bracings will be regarded in the next calculation, see Figure 6. They consist of rods as bracings, circular hollow sections as posts and the beams of the framework as the cords. The rods are subjected for tension forces and only one rod is active at a time.

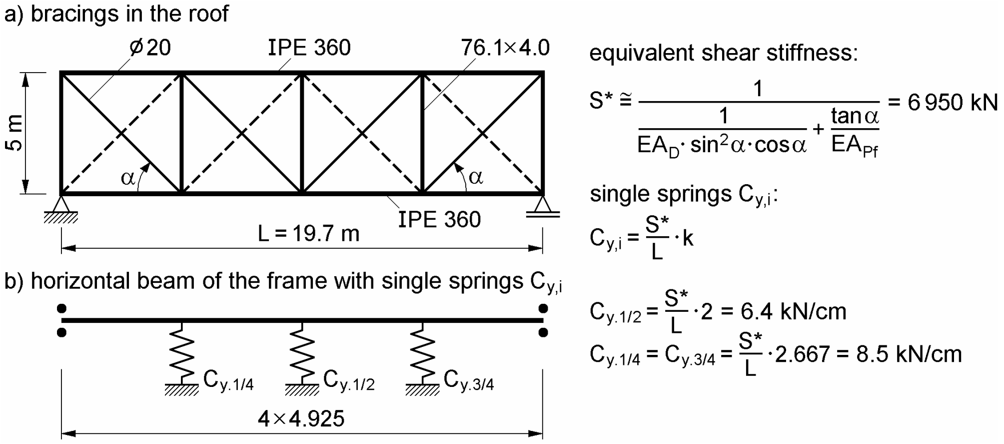


Figure 6 Replacement of the roof bracings by individual single springs

Two roof bracings support five framework beams and two gable walls, see Figure 3. One beam is supported by one third of the shear stiffness of the roof bracing. A method to determine the stiffness of single springs is worked out by Krahwinkel (2001).

Regarding the partial safety factor $\gamma_M = 1{,}1$ the single springs can be determined to 6.4 kN/cm and 8.5 kN/cm. The point of application of the single springs is $z_{VB} = -9$ cm.

The beams of the framework are connected with the columns by end plates and bolts. The end plates act as warping springs at the beam ends and they can be regarded in the verification. The warping springs can be determined for an IPE 360 and an end-plate thickness of $t_p = 25$ mm with:

$$C_{\omega,d} = 17 \cdot 2.5^3 \cdot (36.0 - 1.27) \cdot 8100/(3 \cdot 1.1) = 22.643 \cdot 10^6\ \mathrm{kNcm^3} \qquad (2)$$

The shown supports lead to a critical load factor of $\eta_{Ki,d} = 1.563$ for load case 1 and $\eta_{Ki,d} = 1.489$ for load case 2.

There is a large negative bending moments at the right beam end in load case 2. For this reason haunches are designed at the ends of the beam and these parts are decisive for the verification, see Figure 8. The critical load factor is $\eta_{Ki,d} = 1.489$ (see above) and the corresponding modal shape is shown in Figure 7. The deformations v(x) and ϑ(x) have distinct amplitudes at the right beam end. With the knowledge of the modal shape deformations due to the big negative bending moments at the right beam end lateral torsional buckling is the decisive stability case for this example. The distribution of the deformation v(x) has four waves reducing from the right to the left end. In the area of the horizontal single springs the deformation v(x) is almost zero. That means that the lateral torsional buckling occurs between the horizontal supports and the selected equivalent geometric imperfection in this case includes four waves, shown in Figure 7. The maximum of the displacement is $v_0 = \ell_i/200 = 492.5/200 = 2.47$ cm.

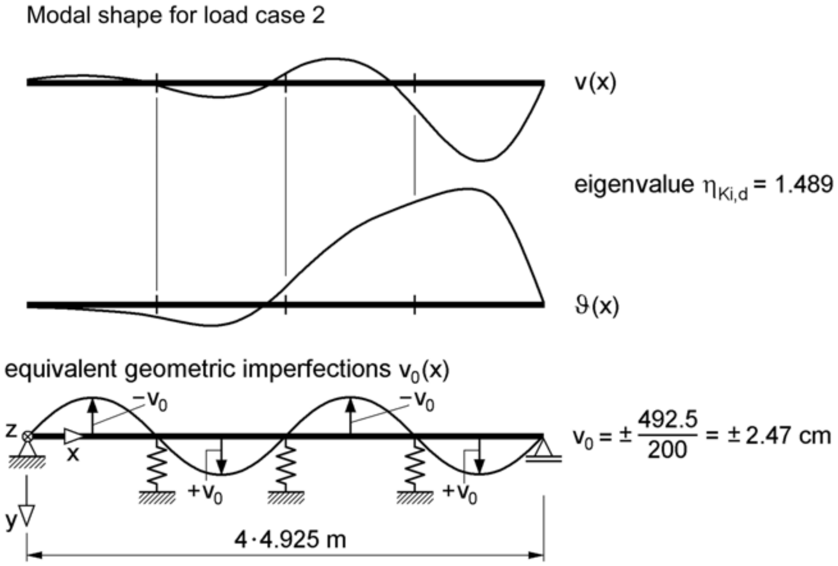


Figure 7 Modal shape of the system in Figure 4c and the equivalent geometric imperfection

For the verification of the load carrying capacity the internal forces can be determined with the program KSTAB and the supports as for instance warping springs at the ends, a continuous torsional spring and single springs can be regarded. Selected results of this verification and the decisive internal forces are shown in Figure 8. The utilization distribution S_d/R_d along the beam is also shown in Figure 8. The partial internal force method by Kindmann (2002) was used for the verification of the plastic cross section capacity.

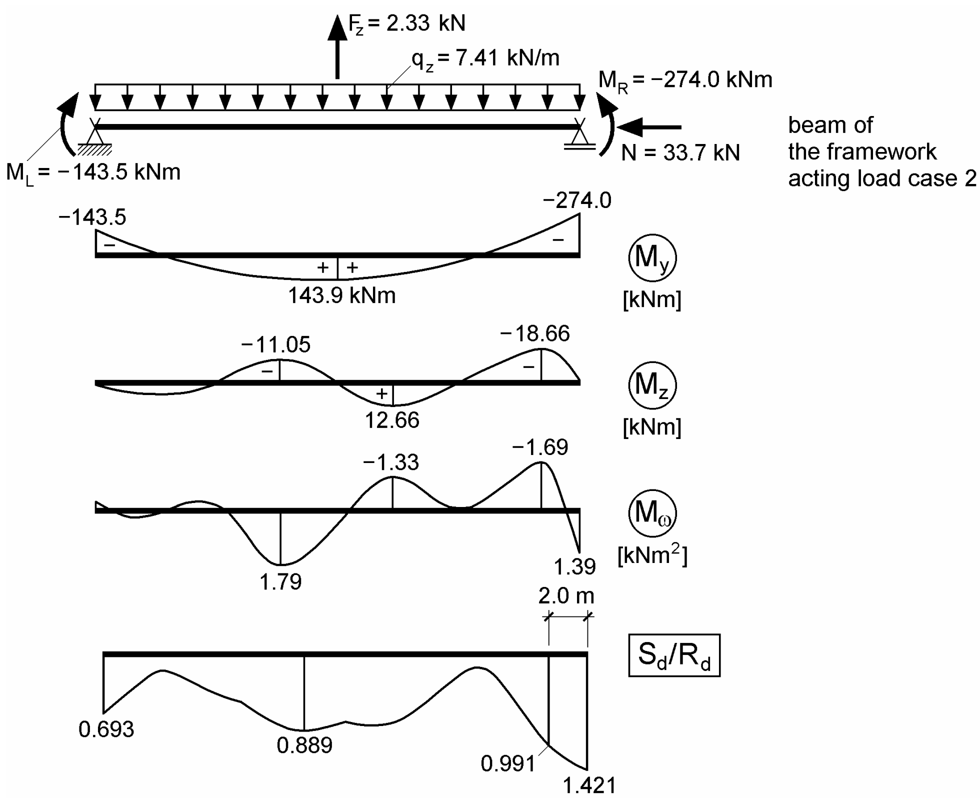


Figure 8 Decisive load case for the verification of the beam of the framework from Figure 4c and utilisation of the cross section S_d/R_d

The used cross section is a hot rolled profile IPE 360. The haunches have to be verified separately because of exceeding at the right beam end up to 42.1 %. In this essay this verification is omitted.

Annotation: The higher stiffness of the beam in the parts of the haunches is neglected. You can also use the method by Linder & Scheer (1998) to determine the continuous torsional spring of the roof cladding. The continuous spring stiffness will increase using this method.

4.3 *Stresses und load carrying behavior of the roof bracing*

Both roof bracings shown in Figure 3 are loaded by horizontal loads which are the wind forces on the gable walls and stresses due to lateral torsional buckling of the beams. Both roof bracings are designed for the compression loads due to wind on the gable walls. The wind loads from the upper half of the gable wall is introduces by the column. In the considered example the wind load can be determined to $W_1 = W_2 = 2 \cdot W_3 = 10.34$ kN . For more information see by Kindmann (2008).

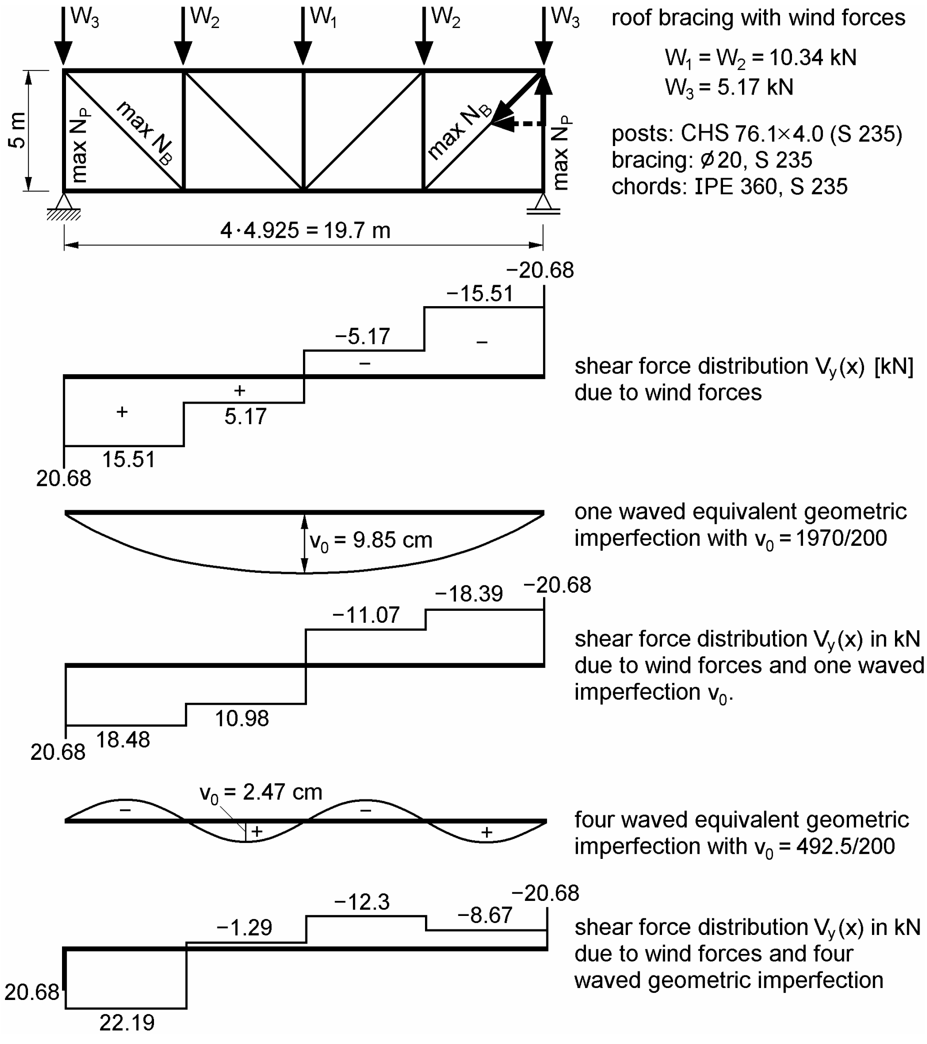


Figure 9 Roof bracing with wind loads, equivalent imperfections and appropriated shear force distributions $V_y(x)$

For the determination of the stresses in the roof bracings the following points will be considered:

- Load case
  The following investigation deals with the load case 1, shown in Figure 4. In this load case the wind blows on the gable walls.
- Supports to stabilize the beam of the framework
  In the verification the all stabilizing supports are regarded, that means the continuous torsional spring (roof topping), the horizontal single springs (bracings) and the warping spring at the ends (end plates).

- Equivalent geometric imperfection
  Figure 9 shows the two regarded equivalent geometric imperfections. The first imperfection is one waved. The one waved distribution corresponds to the assumption in Figure 2 and to the modal shape for a failure of the complete system. Instead of single springs for this system the stiffness of the bracings as equivalent shear field is taken into account. The second imperfection has four waves analogue to the imperfection in Figure 7 for the beam of the framework. In both cases the equivalent geometric imperfection is regarded with $v_0 = \ell/200$. The influences are not decisive, shown in Figure 9.

Only one half of the roof is considered for the determination of the stresses in the bracings. The basic assumptions are shown in Figure 9. For clarity reasons, only the bracings with the wind loads are displayed without the attached frameworks. The equivalent geometric imperfections are applied to each framework beam.

Only one beam is being analyzed regarding all stabilizing supports. The results of the calculation are shown in Figure 9 with the shear force distribution $V_y(x)$. The shown distribution only stresses one roof bracing. Due to the split up of the shear stiffness only one third is associated to the beam. By comparing the shear force distributions shown in Figure 9, the difference of the shear forces between the determination with or without the stresses due to lateral torsional buckling of beams are evident. The maximum of the shear force is $V_y = 22.19$ kN.

Annotation: The shear force at the supports is $V_y = 20.68$ kN in both cases, shown in Figure 9. This shear force is equal to the one for wind loads only. The equivalent geometric imperfections do not lead to forces at the supports due to the equilibrium obtained by $\Sigma F_y = 0$.

5 CONCLUSION

In the present article the load bearing behaviour of a framework beam is shown with the example of Figure 3 and Figure 4. It is not sufficient to just regard horizontal supports for the stabilization of slender beams subjected by positive and negative bending moments, shown in Figure 5. Therefore a continuous torsional spring and warping springs at the beam ends have to be considered. It is shown that all components contributing to the stabilization should be regarded for an economical design of a framework beam.

The second result is, that the brace stresses occurring due to the lateral torsional buckling of the beams depend on the applied distribution of the equivalent geometric imperfection $v_0(x)$, shown in Figure 9. It is the question how equivalent geometric imperfections have to be applied to the beam in order to obtain the decisive stresses in the bracings. It is therefore recommended that depending on the modal shapes more than one equivalent imperfection should be regarded in the investigation. The design can subsequently be carried out using the determined forces due to the stabilization.

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