Stability of I-shaped members with haunches under bending – Design aids for individual member checks

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ABSTRACT: Due to economical reasons, portal frame structures are often designed in a way that the depth of the rafter is increased near the eaves by a haunch to match the load distribution. This paper summarizes the results of a research project on the stability of I-shaped members with haunches under bending. Within the research project extensive parameter studies were performed using a numerical model that was calibrated on experimental tests. As a result, easy to handle design aids for the calculation of the elastic critical buckling moment M_{cr} in form of diagrams and formulas were developed. Additionally, requirements concerning stiffness and arrangement of secondary members are derived that permit to rule out failure due to lateral torsional buckling. These will enable the engineer in daily practice to skip the complex lateral torsional buckling check and to utilise the full resistance of the cross section. Furthermore, rules for cutting out members from complete structures for individual stability checks are specified. As a result of the research project design aids are presented that allow to perform the lateral torsional buckling check of structures with haunches in form of a hand calculation. This is particularly beneficial to small and medium-sized businesses in steel construction that do not perform a finite-element-analysis in daily practice.

1 INITIAL SITUATION AND OBJECTIVES

Portal frame structures made of steel mostly comprise open, I-shaped, hot-rolled sections. Often the depth of the rafter is increased near the eaves by a haunch to match the load distribution. This design offers the economic advantage of a smaller section that can be used for the largest part of the member. In the current standards proof formats are specified for check of out-of-plane stability, that means lateral torsional buckling, that are based on the determination of the elastic critical buckling moment of cut-out members and subsequent determination of the ultimate load by using a buckling curve based on experimental tests and numerical simulations. For the check of uniform members design aids are provided in technical literature. The lateral torsional buckling of haunched members is only covered for special cases and not in a satisfactory way.

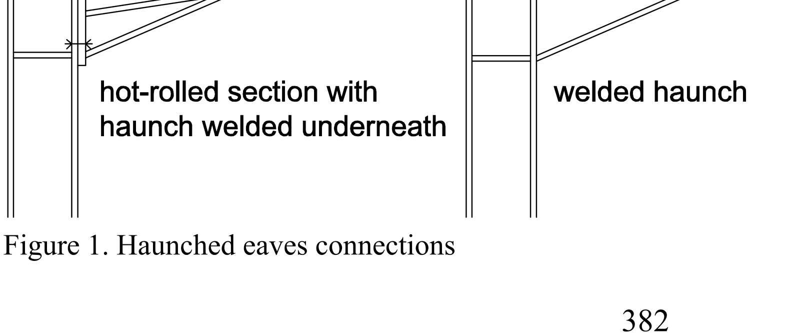


Figure 1. Haunched eaves connections

To analyze the parameters influencing the stability behavior of haunched I-sections through systematic theoretical, experimental and numerical investigations, a research project was carried out. The objective of this project was to establish the fundamentals for creating design aids for the lateral torsional buckling of members with and without haunches. With these design aids a simplified check can be performed by stability assessment of individual members in form of a hand calculation.
The design aids help to close the gap between the simplified procedures and more accurate but complex numerical simulations. Thus, a simplified lateral torsional buckling check based on numerical and experimental results is provided.

2 DESIGN AIDS FOR DETERMINING THE ELASTIC CRITICAL BUCKLING MOMENT

As a result of numerical investigations, design aids for calculating the elastic critical buckling moment for I-sections are developed, taking into account the non-uniform cross-section. For this, complete members with haunches on both sides were analyzed as well as segments between torsional restraints that consist of a haunched part and possibly a part with constant height. In addition, the constructive design of the haunch is varied including welded haunches consisting of three sheets and rolled I-sections with a haunch in form of a halved section welded underneath. As a result, easy to handle design aids in form of both graphical diagrams and formulas have been developed. Below, two examples for these design aids are presented. Assessment diagrams for further cases are found in Ungermann & Strohmann (2009).

2.1 *Welded haunch with torsional restraint at arbitrary location*

The design aids are derived based on the condition that the deformation energy of the haunched member must be equal to that of an imaginary member with uniform but initially unknown properties, see Galéa (1986).

$$\Gamma(h_{eq}, I_{T,eq}) = \Gamma(h(x), I_T(x)) \tag{1}$$

These equivalent properties can be deducted from the energy theorem:

$$\begin{aligned}&\frac{1}{2}\cdot\int_0^L EI_z\cdot\left(\frac{d^2v}{dx^2}\right)^2 dx+\frac{1}{2}\cdot\int_0^L EI_\omega\cdot\left(\frac{d^2\vartheta}{dx^2}\right)^2 dx+\frac{1}{2}\cdot\int_0^L GI_T\cdot\left(\frac{d\vartheta}{dx}\right)^2 dx\\&+\int_0^L M_y(x)\cdot\left(\frac{d^2v}{dx^2}\right)^2\cdot\vartheta dx+\frac{1}{2}\cdot\sum_{k=1}^{p}p_k\cdot d_k\cdot(\vartheta)_{x=xk}+\frac{1}{2}\cdot\int_0^L q\cdot d\cdot\vartheta^2 dx=0\end{aligned} \tag{2}$$

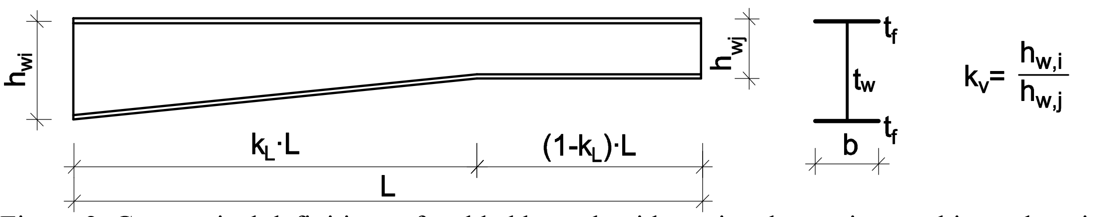


Figure 2. Geometrical definitions of welded haunch with torsional restraint at arbitrary location

Solving (1) for $h_{w,eq}$ and $I_{T,eq}$ and evaluating the formulas thus obtained for different values of k_v and k_L leads to the design aids in Figure 3. With the equivalent section properties the elastic critical buckling moment M_{cr} can be calculated using the familiar formula:

$$M_{cr} = C_1\cdot\frac{\pi^2 EI_z}{L^2}\left[\sqrt{\frac{I_w}{I_z}+\frac{L^2 GI_T}{\pi^2 EI_z}+\left(C_2 z_g\right)^2}-C_2 z_g\right] \tag{3}$$

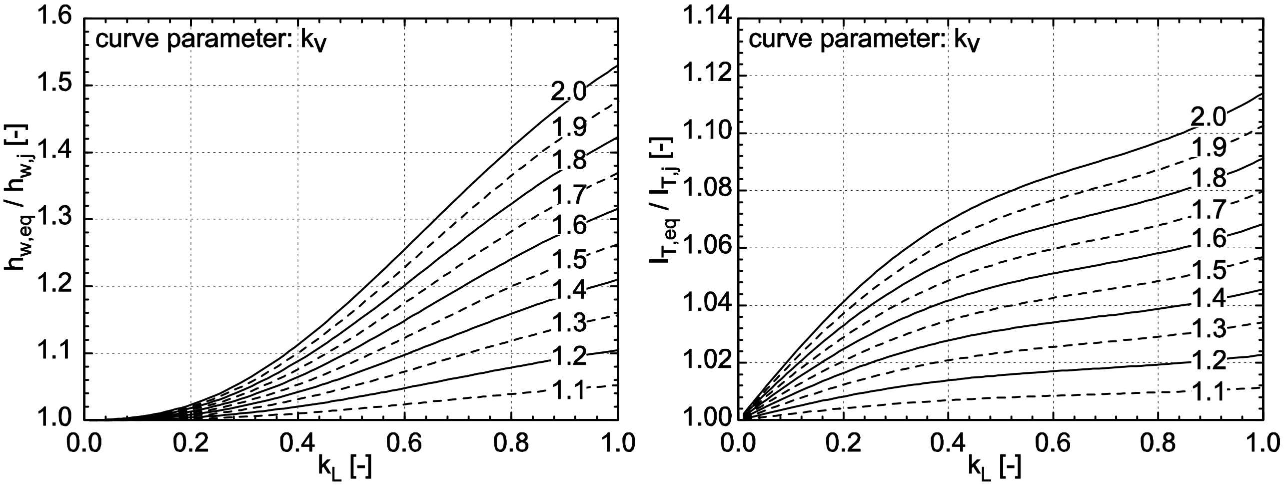


Figure 3. Design aids for determining the equivalent section properties $h_{w,eq}$ and $I_{T,eq}$

2.2 *Rolled section with haunch and torsional restraint at arbitrary location*

By considering the middle flange a noticeable increase of the elastic critical buckling moment M_{cr} compared to the welded design is possible. The numerically determined values for M_{cr} lie between the two extreme cases "original section without haunch" and "length of haunch = member length". Neglecting the lower flange of the original section leads to conservative results.

The objective is therefore to develop design aids that take into account the increase of M_{cr} due to the third flange. The approach using equivalent section properties as in 2.1 already leads to hardly manageable formulas for the special case that the torsional restraint is located at the member end. It is therefore not pursued here.

Instead, a correction factor

$$\xi = \frac{M_{cr,3flan}}{M_{cr,2flan}} \tag{4}$$

is derived. With this the elastic critical buckling moment for the section with three flanges $M_{cr,3flan}$ can be easily calculated by increasing the value $M_{cr,2flan}$ of an equivalent welded section.

$$M_{cr,3flan} = \xi \cdot M_{cr,2flan} \tag{5}$$

The evaluation of ξ has been done in a parameter study. As a result, diagrams for determining the factor ξ were developed. An example for these design aids is shown in Figure 4. This kind of presentation of the results requires one diagram for each end moment ratio ψ. A possible transverse load – represented by the parameter μ – is accounted for in the derivation of the design aids by choosing the minimum values for ξ. However, for the relevant parameter range the overall influence of such a load is small.

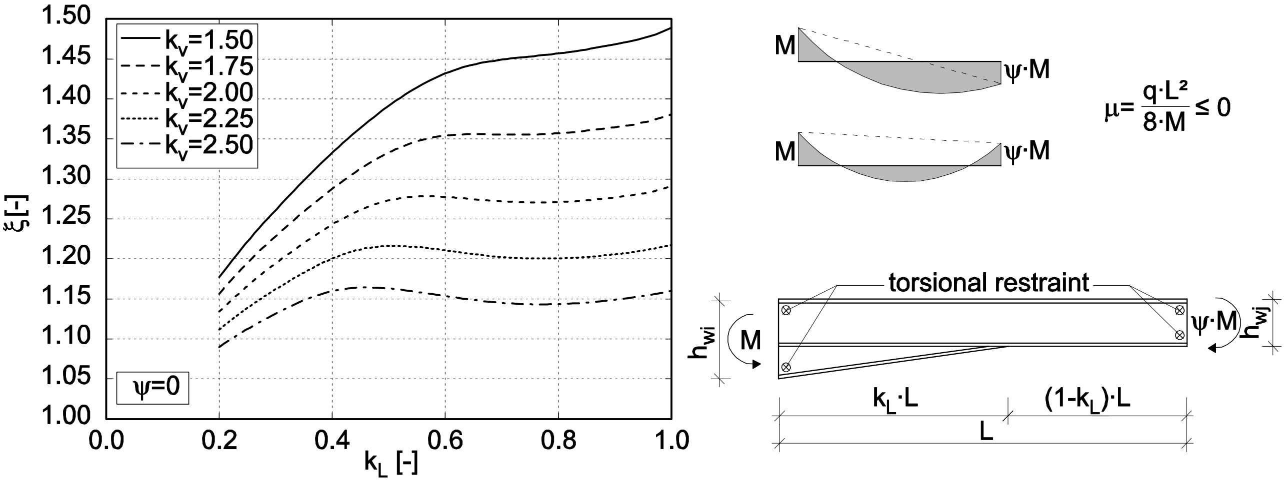


Figure 4. Example of a design aid for determining the equivalent elastic critical buckling moment $M_{cr,3flan}$

3 PREVENTING LATERAL TORSIONAL BUCKLING BY RESTRAINIG DEFORMATIONS

In Lindner (1987a, b) minimum values for sufficient continuous torsional restraint are indicated that are also adopted in the codes DIN 18800 and Eurocode 3. Before expanding the available design aids on haunched members, first the background of the existing relationships is reviewed.

$$M_{cr}^2 = EI_z\left(EI_w\frac{\pi^4}{L^4} + GI_T\frac{\pi^2}{L^2} + c_\vartheta\right) \qquad (6)$$

applies for uniform members exposed to a constant moment distribution. In the deduction

$$\chi_{LT} = \left(\frac{1}{1+\bar{\lambda}_{LT}^5}\right)^{0,4} \qquad (7)$$

is applied as the appropriate buckling curve for lateral torsional buckling. Setting L = ∞ and assuming that the moment resistance of the member has to reach at least 95% of the moment resistance of the cross-section leads to a minimum elastic critical buckling moment of:

$$M_{cr*} = 2.216 \cdot M_{pl} \qquad (8)$$

Placing (6) in (8) and solving for c_ϑ leads to the required continuous torsional restraint:

$$c_\vartheta \geq \frac{(2.216 \cdot M_{pl})^2}{EI_z} \qquad (9)$$

For non-constant moment distribution this results in:

$$c_\vartheta \geq \frac{(2.216 \cdot M_{pl})^2}{\zeta^2 \cdot EI_z} \qquad (10)$$

This relationship can be found in DIN 18800 and Eurocode 3 in the form

$$c_{\vartheta,k} \geq \frac{M_{pl,k}^2}{EI_z} \cdot K_\vartheta \cdot K_v \qquad (11)$$

with K_v = 1,0 for plastic analysis
K_ϑ = factor for considering the moment distribution

The varying moment distributions are accounted for by the factor K_ϑ which has been derived by evaluation of (10) and is included in Eurocode 3-1-1, table BB.1. The equation (11) including the coefficients K_ϑ has been transferred from DIN 18800 to Eurocode 3 although the lateral torsional buckling curve (7) that is the basis of the derivations is not valid there. Strictly speaking, when using the modified lateral torsional buckling curves for rolled or equivalent welded sections under consideration of the modification factor f, the required torsional restraint has to be calculated by

$$\text{buckling curve b}(h/b \leq 2):\quad c_\vartheta \geq \frac{(2,56 \cdot M_{pl})^2}{\zeta^2 \cdot EI_z}$$

$$\text{buckling curve c}(h/b > 2):\quad c_\vartheta \geq \frac{(3,17 \cdot M_{pl})^2}{\zeta^2 \cdot EI_z} \qquad (12)$$

This discrepancy has to be rated within an evaluation of the appropriate lateral torsional buckling curves. For the following derivations, the condition (11) is applied in agreement with Eurocode 3, Annex BB.2.2.

The required torsional restraint to achieve M_{cr*} depends strongly on the moment distribution. First, a systematic evaluation of K_ϑ for different moment distributions is performed. The existing tables only cover the special case where the absolute value of the moment at the support and at midspan is equal, i.e. μ=-2.0. For this case K_ϑ is 3.5.

A parameter study was performed with the software LTBeam to calculate the torsional restraint c_ϑ which is necessary in order to achieve M_{cr*}. Then, the condition (11) is solved for K_ϑ and the results are plotted over the load parameter μ. On the left side of Figure 5 the numerical results (black curves) are compared to the theoretical values (grey curves) which is defined by

$$c_\vartheta \geq \frac{M_{pl}^2}{EI_z} \cdot \underbrace{\left(\frac{2.216}{\zeta}\right)^2}_{K_\vartheta} \tag{13}$$

with

$$\zeta = f(\mu) = \frac{M_{cr,M\neq const.}}{M_{cr,M=const.}} \tag{14}$$

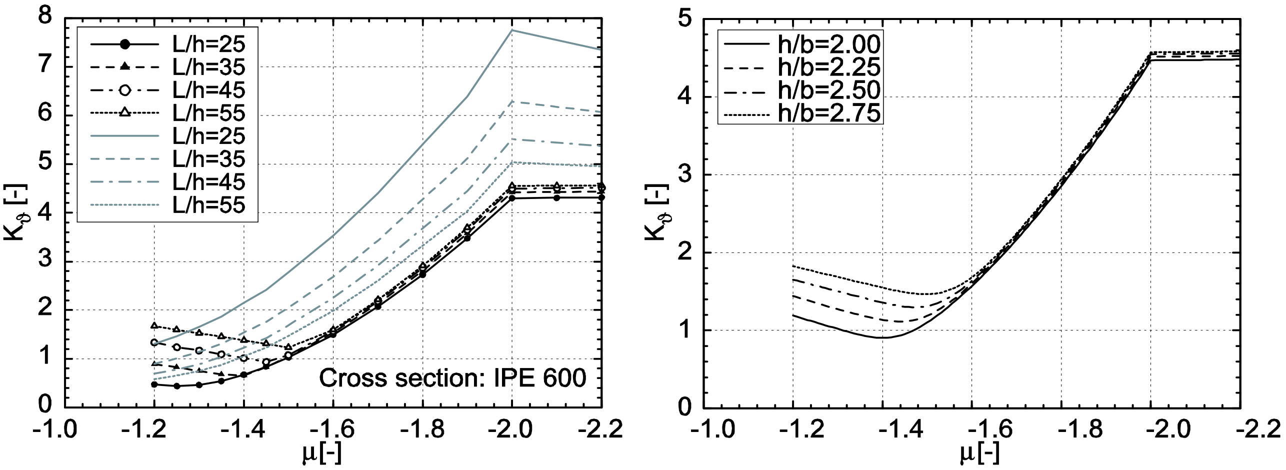


Figure 5. Comparison of numerical and theoretical values for K_ϑ (left) and design aid (right)

In this case, ζ is calculated backwards from the numerically obtained value for M_{cr}. ζ gets larger with increasing member length. The limit value of μ for practically relevant cases is -1.5 which corresponds to a member with fixed end conditions. For uniform members larger μ-values can not be achieved, yet the full parameter range is analyzed for the subsequent expansion on haunched members. The following results could be observed: In the relevant range of μ the theoretical value

$$K_\vartheta = \left(\frac{2.216}{\zeta}\right)^2 \tag{15}$$

is always on the safe side. The stockier a member is the more conservative are the results. Compared to the numerically calculated results the K_ϑ-value of 3.5 that is specified in the codes is on the unsafe side. This is due to the fact that in the derivation in equation (10) the maximum value of ζ was used instead of the minimum.
In the right sector of the diagram there is nearly no dependency of K_ϑ on the member length and cross-section. The dependency of K_ϑ on the cross-section can be expressed with the help of the height to width ratio of the member. As a result, the diagram on the right side of Figure 5 for the determination of K_ϑ was derived.
In the next step, the parameter study was expanded to non-uniform members. For non-uniform members, instead of M_{pl} – that is variable along the member's length – the minimum load amplifier to reach the cross-section resistance α_{pl} has to be considered.

$$\alpha_{pl} = \max \frac{M_{pl}(x)}{M(x)} \tag{16}$$

Placing (16) in (11)

$$M_{pl,k}^2 = (\alpha_{pl} \cdot \max M)^2 \tag{17}$$

leads to

$$c_{\vartheta,k} \geq \frac{(\alpha_{pl} \cdot \max. M)^2}{EI_z} \cdot K_\vartheta \tag{18}$$

The determination of K_ϑ was again performed by calculating the required torsional restraint c_ϑ with the software LTBeam and solving equation (18) for K_ϑ.
As a result of the parameter study it could be detected that the design of the haunch – welded section or rolled section with haunch welded underneath – has a much greater influence on K_ϑ than the geometry of the haunch, expressed through k_v and k_L. Therefore, for the design proposal the maximum K_ϑ-values were evaluated depending on the moment distribution only.
In Figure 6 the factor K_ϑ is presented for rolled sections with a haunch welded underneath depending on the load parameter μ. An analog diagram for the welded haunch can be found in Ungermann & Strohmann (2009). With the help of these diagrams and equation (18) the required torsional restraint for haunched members can be calculated. For uniform members (18) passes into the well-known formula from the standards.

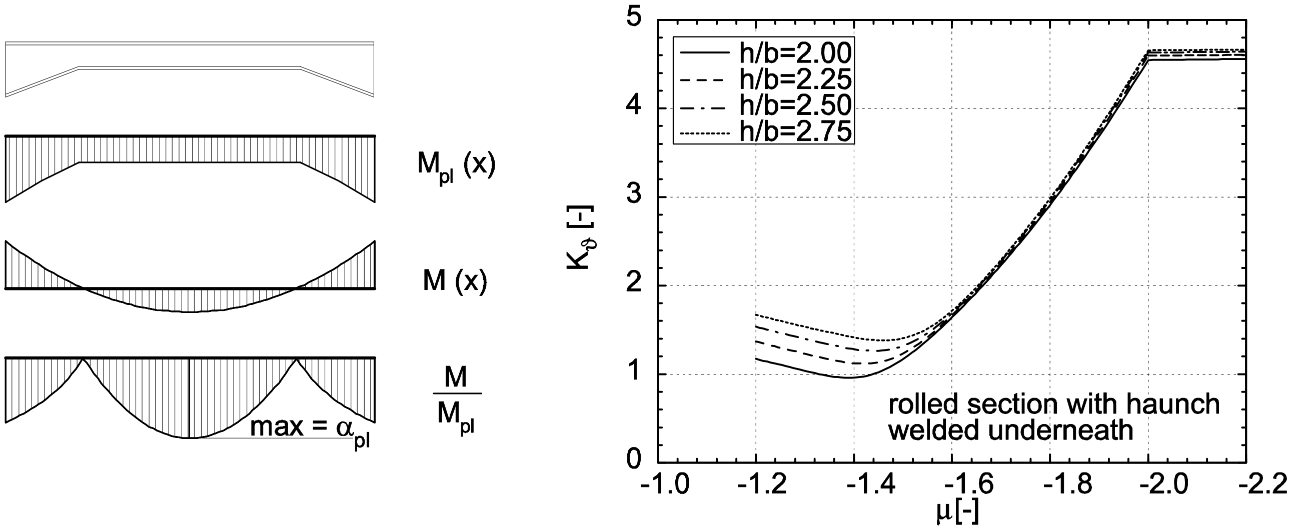


Figure 6. Definition of α_{pl} (left) and design aid for determining the required stiffness of torsional restraint

4 CUTTING OUT MEMBERS FROM COMPLETE STRUCTURES FOR INDIVIDUAL STABILITY CHECKS

In this section, rules for cutting out individual members from complete frame structures are indicated. Both the existing and the new design aids that are described above require “fork conditions” at the member’s ends. At first, the required minimum stiffness of discrete torsional restraints that will lead to a failure of the member between the restraints is derived. Then requirements for the structural design of the frame corners are specified that ensure full coaction of column and beam. If complied with these rules, the stability assessment using individual cut-out members will be on the safe side.

4.1 *Minimum stiffness of discrete torsional restraints to exact failure between the restraints*

The design aids for the minimum stiffness of discrete torsional restraints are derived under the assumption that the elastic critical buckling moment of a member with fork conditions at the position of these restraints

$$M_{cr}^2 = EI_z \left(EI_w \frac{\pi^4}{(L/(n+1))^4} + GI_T \frac{\pi^2}{(L/(n+1))^2} \right) \tag{19}$$

has to be equal to the elastic critical buckling moment of the same member with continuous torsional restraint:

$$M_{cr}^2 = EI_z\left(EI_w\left(\frac{\pi}{L}\right)^4 + GI_T\left(\frac{\pi}{L}\right)^2 + \frac{n\cdot c_{\vartheta,dis}}{L}\right) \tag{20}$$

For members with uniform cross-section and uniform moment distribution, equalizing the equations (20) and (19) and solving for $c_{\vartheta,dis}$ leads to:

$$c_{\vartheta,dis} \geq \frac{L}{n}\left[EI_w\left(\frac{\pi}{L}\right)^4 (n+1)^4 + GI_T\left(\frac{\pi}{L}\right)^2 (n+1)^2\right] \tag{21}$$

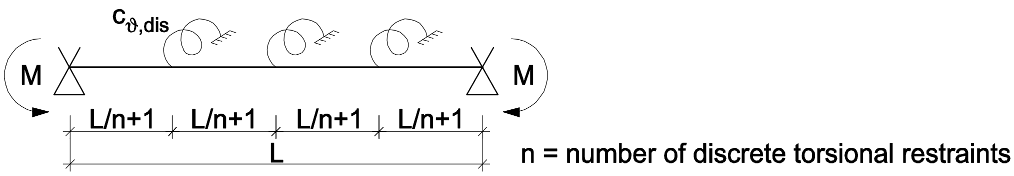


Figure 7. Member with discrete torsional restraints – definitions

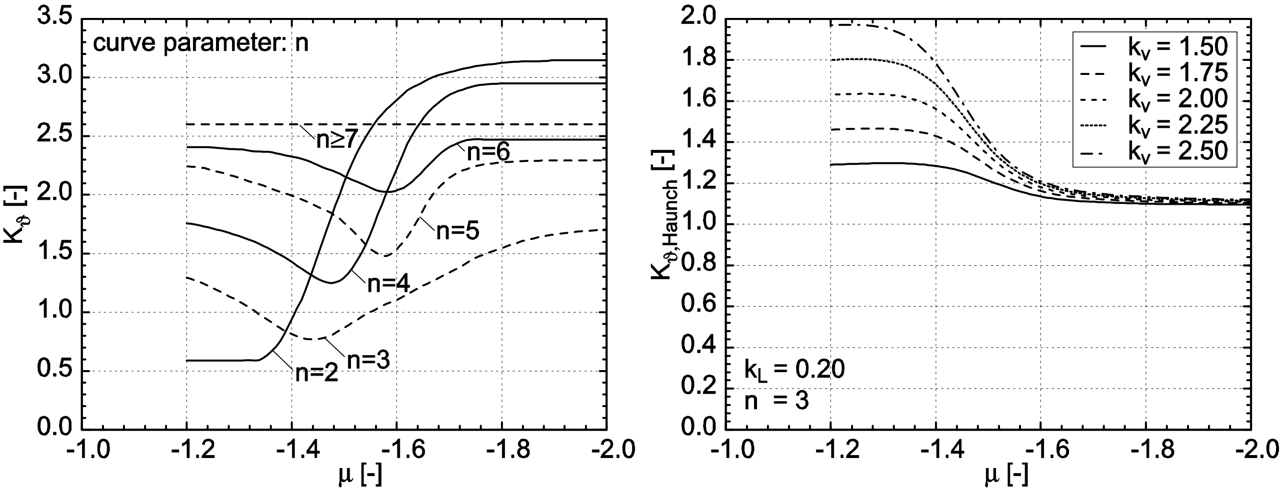


Figure 8. K_ϑ-factors for calculating the miminum stiffness of discrete torsional restraints

For non-uniform members and moment gradients, the required stiffness of the discrete torsional restraints has been calculated in a parameter study. As a result, correction factors K_ϑ and $K_{\vartheta,Haunch}$ have been derived according to Figure 8. With these the required stiffness of the torsional restraints can be calculated:

$$c_{\vartheta,dis} \geq K_\vartheta \cdot K_{\vartheta,Haunch} \cdot \frac{L}{n}\left[EI_w\left(\frac{\pi}{L}\right)^4 (n+1)^4 + GI_T\left(\frac{\pi}{L}\right)^2 (n+1)^2\right] \tag{22}$$

4.2 *Requirements for the structural design of the frame corners*

The requirements for the out-of-plane stability of the frame corner result from the assumption that lateral torsional buckling of column and rafter are the controlling failure mode and popping out of the inner corner as shown in Figure 9 is prevented. For this, buckling analyses of complete frames were performed under variation of the structural design of the frame corner taking into account torsional restraints of the rafter. The elastic critical buckling loads were then compared to the values obtained for individual members cut out of the complete structure.

The result is a classification of the investigated corner types depending on the degree of torsional restraint of the rafter. In Table 1 the configurations for which M_{cr} of the controlling individual member can be achieved are marked with two ticks. For combinations with only one tick an additional analysis is required. In this case, M_{cr} of the controlling individual member can only be achieved if the critical load of the rafter is lower than the critical load of the column. For all other combinations (-) buckling of the inner corner is the controlling failure mode and therefore an assessment of the structure using cut-out members is on the unsafe side.

It becomes evident that the requirements for the corner design do not necessarily lead to increased welding effort and labor costs. In most cases, one stiffener located in the tension area of the corner is sufficient.

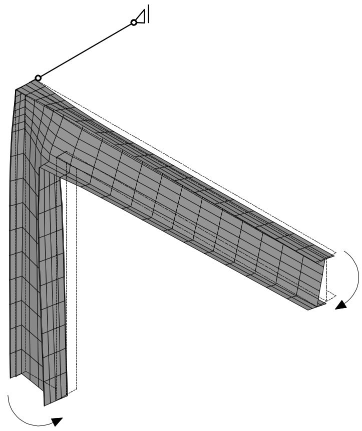
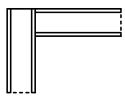
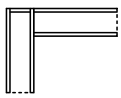
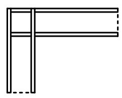
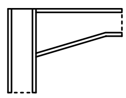
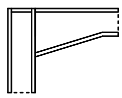
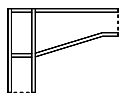
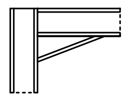
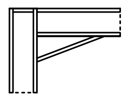


Figure 9. Frame corner without stiffeners – failure mode: Buckling of the inner corner

Table 1. Classification of corner types – excerpt

								
Lateral restraints at the corners	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓	✓✓
n=2	✓	✓✓	✓✓	✓	✓✓	✓✓	✓✓	✓✓
n=3	-	✓✓	✓✓	-	✓✓	✓✓	✓	✓✓
n=4	-	✓✓	✓✓	-	✓✓	✓✓	✓	✓✓
n=5	-	✓✓	✓✓	-	-	✓✓	✓	✓✓
n=6	-	✓✓	✓✓	-	-	-	✓	✓✓
Continuous torsional restraint	-	✓✓	✓✓	-	✓✓	✓✓	-	✓✓

5 SUMMARY AND CONCLUSION

The design aids developed within the research project summarized here present the exact results of numerical calculations in a revised form for use with the equivalent column method. Thus, a lateral torsional buckling check that is based on numerically and experimentally obtained results is offered. This is particularly beneficial to small and medium-sized businesses in steel construction that do not perform a finite-element-analysis in daily practice.

ACKNOWLEDGEMENT

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