Ultimate and serviceability limit state optimization of cold-formed steel hat-shaped beams

D. Honfi
Division of Structural Engineering, Lund University, Lund, Sweden

ABSTRACT: Cold-formed steel hat-shaped beams are commonly used in light-weight steel construction systems, such as residential housing structures. The ultimate strength of these sections is affected by their complex stability behavior (e.g. local, distortional and global buckling), therefore an advanced design methodology is needed to calculate the load bearing resistance in the ultimate limit state. On the other hand the limiting requirements in the design of such structural elements are often the serviceability provisions of the applied design code, which are investigated under the serviceability limit state load conditions. The aim of the present study is to find the optimal cross-sectional dimensions for cold-formed steel hat-profiles with various supporting conditions according to the Eurocode.

1 INTRODUCTION

1.1 *Background*

The various types of cold-formed steel sections with their flexibility in the fabrication process have a huge demand of optimization for different structural functions (struts, purlins, studs, etc.). The difficulties of the design of these sections, due to their complex stability behavior – e.g. local, distortional and global buckling – require an advanced design methodology (Yu 2000).

A genetic algorithm (GA) optimization procedure – based on the Eurocode prescriptions (CEN 2005) – has been shown by Honfi (2006) finding the optimum cross-sectional dimensions for hat shaped beams focusing only on the ultimate limit state (ULS).

However the limiting requirements in the design of such structural elements are often the serviceability provisions of the applied design code, which are investigated under the serviceability limit state (SLS) load conditions.

The aim of the present study is to find the optimal cross-sectional dimensions for cold-formed steel hat-profiles with various supporting conditions according to the European standard, taken into account both – strength and serviceability – requirements.

1.2 *Structural optimization*

The structural optimization is defined mathematically as (Kirsch 1993):

$$Z = f(X) \rightarrow \min, \quad (1)$$

$$g_j(X) \leq 0 \quad j = 1,\ldots,n_g, \quad (2)$$

$$h_j(X) = 0 \quad j = 1,\ldots,n_h, \quad (3)$$

where X is the vector of design variables (material data, topology, geometry, cross-sectional data, etc.), Z is the objective function (weight, cost), $g_j(X) \leq 0$ and $h_j(X) = 0$ are the inequalty and equality constraints, respectively. During the optimization process those design variables in the design space are to find, which minimize the objective function.

1.3 *Genetic algorithm*

The genetic algorithm is a useful tool for solving optimization problems based on the theory of natural selection and can be applied in the design of cold-formed steel beams (Lu 2003). The algorithm begins with an initial population, which represents a set of solutions. At each step of the genetic algorithm the individuals (chromosomes) of the current population are selected (selection) and modified (crossover, mutation) to produce the next population to evolve toward the optimal solution. The chromosomes are selected according to their fitness to produce the next population. In the current study the chromosomes are the cross-sectional dimensions of the hat shaped beams as presented in section 3.

The main steps of a typical genetic algorithm are the followings (Obitko 1998):

1. Generate initial population of n chromosomes (suitable solutions for the problem).
2. Evaluate objective function (*fitness*) *f(x)* of each chromosome x in the population.
3. Create a new population by repeating following steps until the new population is complete.
4. Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to be selected).
5. With a *crossover* probability generate offspring from the two parents.
6. With a *mutation* probability mutate offspring.
7. Place offspring in the new population.
8. Use the new population for a further run of the algorithm.
9. If the end condition is satisfied, stop, and return the best solution in current population.
10. Repeat from step 2.

Reaching the stopping criteria the final population will contain the optimal solution, hopefully the global optima of the problem.

2 COLD-FORMED HAT-SHAPED BEAMS

2.1 *Structural system and behaviour*

In the previous research project an experimental program is developed covering the structural systems applied by Lindab Ltd (Honfi & Dunai 2006). The aim of the experimental investigation of the hat-shaped element was the determination of the structural behavior and the resistance of the member to verify the finite element model. However another purpose of the experiments was to investigate the effect of web-openings in such sections, this fact is not relevant in the current paper. The applied statical models are presented in Figure 1, such as simply supported system (*sys1*), simply supported system with short (600 mm) and long (1000 mm) cantilever beam (*sys2* and *sys3* respectively) and continuous system (*sys4*) with two-spans. The span length is 1275 mm for all cases. All systems are loaded uniformly distributed.

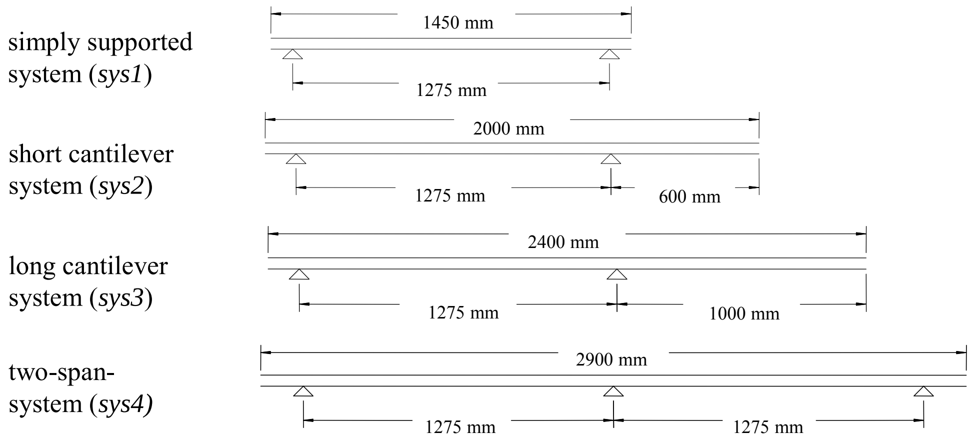

Figure 1. Investigated statical systems.

The experimental and numerical investigations showed that the typical failure modes for the mentioned statical systems, as follows:

1. Simply supported system (*sys1*): local buckling due to bending in the middle of the span.
2. Cantilever systems (*sys2* and *sys3*): local buckling at the support of the cantilever side due to the interaction of bending and transverse force (web crippling).
3. Two-span-system (*sys4*): local buckling at the mid-support due to the interaction of bending and transverse force. After forming a plastic mechanism at the support, further loading could be applied, until bending failure occurred at the mid-span(s).

The cross-sectional optimization is carried out for the above mentioned statical systems considering the described failures and the deflection limits.

2.2 *Strength calculation according to Eurocode*

The calculation of cold-formed steel structural members according to EC3 is more complicated than those are fabricated by hot-rolling. The supplementary rules are given in EN 1993-1-3 (CEN 2005). The code uses the effective width approach to take into account the effect of local and distortional buckling which are the characteristic failure modes of these members. The strength reduction due to local buckling with the effective width of each element, due to distortional buckling with a reduced thickness of the given element is considered. To obtain the load bearing capacity, q_{Rd} of a hat-shaped beam the following criteria must be satisfied according to the bending moments and the transverse forces (reactions):

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0 \text{- in the spans and at the supports,} \qquad (4)$$

$$\frac{F_{Ed}}{R_{w,Rd}} \leq 1.0 \text{ and } \frac{M_{Ed}}{M_{c,Rd}} + \frac{F_{Ed}}{R_{w,Rd}} \leq 1.25 \quad \text{- at the supports.} \qquad (5)$$

In the Equations 4-5 M_{Ed} and F_{Ed} are the applied bending moment and transverse force respectively, furthermore $M_{c,Rd}$ and $R_{w,Rd}$ denote the bending moment resistance and the local transverse resistance of the web. The calculations are carried out assuming uniformly distributed load.

2.3 *Deflection limits according to Eurocode*

The vertical deflections of horizontal structural elements should be limited to avoid deformations that affect appearance/comfort/functioning of the structure or that cause damage to finishes or non-structural members.

The National Annexes (NA) of EN 1993 may specify the limits of deflections. The values given in the NAs are only suggested values, there are not obligatory rules given.

The applied deflection limit in the current study is $L/250$ for beams and the half of this value at the end of cantilevers, where L is the span or the length of the cantilever respectively. The deflections have been calculated using elastic theory from the effective cross-section as it is prescribed in the code.

3 OPTIMIZATION BY GENETIC ALGORITHM

In the current study hat-shaped cold-formed beams with different statical systems are investigated by EC3 requirements. The design variables – the height of the section, h, width of the flange, b, the depth of the lip, c and the angle of the web α – are presented in Figure 2. Other parameters used for the calculation, such as thickness, the internal radius of the corners and the material properties are fixed during the optimization.

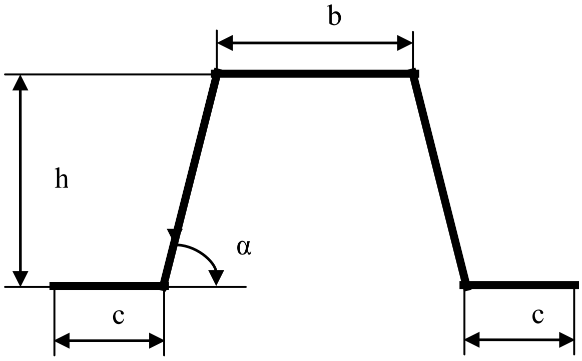

Figure 2. Dimensions of the cross-section.

The current optimization problem is defined as:

$$Maximize \ \frac{q_{Rd}}{A_g}, \tag{6}$$

where q_{Rd} is the load bearing capacity of the member (the maximum load which can be applied without violating the strength and deflection constraint) and A_g is the gross cross-sectional area, subjected to geometrical constraints specified in the standard and given in Equation 7 (CEN 2005). Since A_g is proportional to the weight q_{Rd}/A_g represents the load bearing efficiency of a given section.

$$\frac{h}{\sin\alpha \cdot t} \le 500,\ \frac{b}{t} \le 60,\ \frac{c}{t} \le 50,\ 0.2 \le \frac{c}{h/\sin(\alpha)} \le 0.6 \text{ and } 45° \le \alpha \le 90° \tag{7}$$

To transform the problem into an unconstrained one the objective function is changed by integrating the penalty functions according to these constraints.

$$Maximize\!: \ F = \begin{cases} \dfrac{q_{Rd}}{A_g} - P & if \ \dfrac{q_{Rd}}{A_g} > P, and \\ 0 & otherwise, \end{cases} \tag{8}$$

where P represents penalty function given by the constraint violations.
The optimization is carried out with the Matlab Genetic Algorithm and Direct Search Toolbox (The MathWorks 2004). For encoding the binary string encoding is used. The selection method is rank selection, for crossover the so called scattered crossover is chosen. A random binary vector is generated and, where the vector is 1, the genes are selected from the first parent, and where the vector is 0 the genes are selected from the second parent. The crossover rate is set to 0.8 and mutation rate is to 0.01.
Each population contained 300 individuals and 10 generations are generated. Each GA performed 10 runs, and the best result is considered as the optimal solution.

4 RESULTS

Since the aim of the investigation is to study the effect of implementing the serviceability criteria in the optimization process some parameters are fixed. The thickness of plate from which the beams are manufactured is 0.7 mm, and the applied steel quality is S235.
When investigating the ULS and SLS simultaneously, one should pay attention to the fact, that the strength and the serviceability of the structure should be checked at different load levels. It means, that the ratio of the ultimate load to the service load influences the actual design. This ratio is determined by the safety factors and combination factors and is different for various design situations.
Since the current optimization focuses on the efficiency of the elements - the load carrying capacity of a single member should be maximized – there is no effect of the mentioned ratio on the optimum section for a given structural system. As Table 1 illustrates, the load ratio ($LR=q_{ULS}/q_{SLS}$) does not influence the dimensions of the optimal cross-section for – in this case – the simple supported system (*sys1*). In the upper half of the table the results for ULS design are presented, while in the lower half those, which are calculated considering only the SLS. The columns h, b, c and α represent the previously discussed geometrical dimensions of the cross-section. The values of $q_{ULS,Rd}$, $q_{SLS,Rd}$ and q_{Rd} present the maximum applicable ultimate and service load of a single beam and the load carrying capacity (the minimum of the previous two) respectively. The ultimate load is divided by the load factor (LR) to make it comparable to the service load (therefore the numerical values are different for various load ratios). In the last column the load carrying capacity is divided by the cross-sectional area to show the efficiency of a given section.

Table 1. Optimization results for the simply supported system (*sys1*)

sys	opt	LR	h [mm]	b [mm]	c [mm]	α [°]	$q_{ULS,Rd}$ [kN/m]	$q_{SLS,Rd}$ [kN/m]	q_{Rd} [kN/m]	Area [mm^2]	q_{Rd} /A [kN/m/mm^2]
1	ULS	1.1	41	25	15	90	1.42	0.75	0.75	95.9	0.0078
1	ULS	1.2	41	25	15	90	1.30	0.75	0.75	95.9	0.0078
1	ULS	1.3	41	25	15	90	1.20	0.75	0.75	95.9	0.0078
1	ULS	1.4	41	25	15	90	1.11	0.75	0.75	95.9	0.0078
1	ULS	1.5	41	25	15	90	1.04	0.75	0.75	95.9	0.0078
1	SLS	1.1	41	34	24	90	1.60	0.92	0.92	114.8	0.0080
1	SLS	1.2	41	34	24	90	1.47	0.92	0.92	114.8	0.0080
1	SLS	1.3	41	34	24	90	1.36	0.92	0.92	114.8	0.0080
1	SLS	1.4	41	34	24	90	1.26	0.92	0.92	114.8	0.0080
1	SLS	1.5	41	34	24	90	1.18	0.92	0.92	114.8	0.0080

The optimization for the SLS gives even more uniform solution. Since the beam was considered to be elastic and the deflections are calculated from the effective cross-section, the efficiency in this case depends only on the second moment of inertia of the effective section:

$$q_{SLS,Rd} = f(I_{eff}) \tag{9}$$

which means that the optimization problem in that case is given by:

$$Maximize\left(\frac{I_{eff}}{A_g}\right) \tag{10}$$

This indicates that the optimum cross-section is independent from the deflection limit too. The same calculation has been carried out for the limit L/300 and the results are the same.
Taken into account the ULS and SLS in the optimization simultaneously in case of *sys1* (simply supported beam) and *sys3* (long cantilever) the deflection criteria is decisive, while for the other two systems the strength criteria decides.
The results are summarized in Table 2 taken into account only the ultimate or the serviceability limit state or both of them. The numerical values in the table belong to the case LR=1.1, however the optimal sections are the same in every other case. The results suggest that it would be enough to produce 2 different types of cross-sections to cover these structural systems with an efficient product family (from a given raw material). It is not mentioned before, but obviously the angle α equals 90° in all cases, since the flange and the lips should be as far as possible from each other to achieve a large section modulus and second moment of inertia.

Table 2. Summarized results of optimization (*LR*=1.1)

sys	opt	h	b	c	α	$q_{ULS,Rd}$	$q_{SLS,Rd}$	q_{Rd}	Area	q/A
		[mm]	[mm]	[mm]	[°]	[kN/m]	[kN/m]	[kN/m]	[mm^2]	[kN/m/mm^2]
1	ULS	41	25	15	90	1.42	0.75	0.75	95.9	0.008
2	ULS	41	20	10	90	1.07	1.32	1.07	85.4	0.013
3	ULS	41	29	19	90	0.51	0.18	0.18	104.3	0.002
4	ULS	41	20	10	90	0.95	3.09	0.95	85.4	0.011
1	SLS	41	34	24	90	1.60	0.92	0.92	114.8	0.008
2	SLS	41	34	24	90	1.35	1.97	1.35	114.8	0.012
3	SLS	41	34	24	90	0.56	0.20	0.20	114.8	0.002
4	SLS	41	34	24	90	1.19	4.62	1.19	114.8	0.010
1	both	41	34	24	90	1.60	0.92	0.92	114.8	0.008
2	both	41	20	10	90	1.07	1.32	1.07	85.4	0.013
3	both	41	34	24	90	0.56	0.20	0.20	114.8	0.002
4	both	41	20	10	90	0.95	3.09	0.95	85.4	0.011

5 CONCLUSIONS

Thin-walled hat-shaped beams have been optimized by a genetic algorithm taken into account the ultimate strength and serviceability restrictions of Eurocode. It has been found that the optimum cross section in means of load bearing efficiency is independent from the relation between the ultimate and serviceability load combinations and is not affected by the deflection limit.
For a given structural system only one optimal cross-section can be defined, what is promising from a production point of view.

REFERENCES

Yu, WW (2000) *Cold-formed steel design.* New York: John Wiley & Sons.

Honfi, D (2006) Neural network based genetic algorithm optimization of hat-shaped beams. *Proceedings of the Fifth International Conference on Engineering Computational Technology.* Stirlingshire: Civil-Comp Press, Paper 52.

Honfi, D & Dunai, L (2006) Experimental and numerical investigation on hat-shaped beam members with hole in the web. *International colloquium on stability and ductility of steel structures.* Lisbon, Portugal.

Kirsch, U (1993) *Structural optimization, Fundamentals and applications.* Heidelberg: Springer-Verlag.

Lu, W (2003) Optimum design of cold-formed steel purlins using genetic algorithms. *Helsinki University of Technology, Laboratory of Steel Structures*. Publications 25.

Obitko, M (1998) Introduction to genetic algorithms. Http://cs.felk.cvut.cz/~xobitko/ga/.

CEN (2005) EN 1993-1-3 Eurocode 3: Design of steel structures – Part 1-3: General rules – Supplementary rules for cold-formed thin gauge members and sheeting.

The MathWorks (2004) Matlab Genetic Algorithm and Direct Search Toolbox User's guide. Http://www.mathworks.com.