Reliability of steel flexural members according to EC in serviceability limit state

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ABSTRACT: To achieve a relatively consistent probability of failure for structural elements, most design codes apply reliability based code calibration process. Such approaches commonly focus on the ultimate strength of the structural members, which is related to the ultimate limit state (ULS). In the design of steel beams the performance of the structural elements is often limited by the serviceability requirements, which are related to the serviceability limit state (SLS) using different load combinations than those applied in the ultimate limit state. The current study aims to investigate the reliability for serviceability design for flexural steel members according to the specifications of the Eurocode. Second-order reliability method (SORM) is applied to determine the reliability index for different load ratios.

1 INTRODUCTION

1.1 *Serviceability*

Ultimate failure of structural elements is relatively rare. However, serviceability non-compliances occur more frequently. The main type of serviceability non-compliances are excessive floor and roof deflections which may cause for example (Stewart 1996):

- cracking and local crushing of structural and non-structural masonry walls;
- cracking of reinforced concrete floor beams and slabs;
- gaps below partitions;
- noticeable dishing of floors;
- doors out of square;
- filing cabinets and desks tilted;
- cracking of plastered ceilings;
- damage to services;
- ponding and water/moisture penetration (in case of roof);
- or simply be aesthetically annoying or give the feeling of being unsafe.

To achieve a relatively consistent probability of failure for structural elements, most design codes – such as Eurocode – apply reliability based code calibration process. Such approaches commonly focus on the ultimate strength of the structural members. Therefore the characteristic values of actions

and the combination factors – which are used in serviceability limit states (SLS) as well – are mainly developed and optimized for the ultimate limit state (ULS).
The present paper aims to investigate the reliability of Eurocode specifications for serviceability, considering a steel structural member subjected to bending.

2 DESIGN ACCORDING TO EUROCODE

2.1 *Deflection limits*

The vertical deflections of horizontal structural elements should be limited to avoid deformations that affect appearance/comfort/functioning of the structure or that cause damage to finishes or non-structural members.

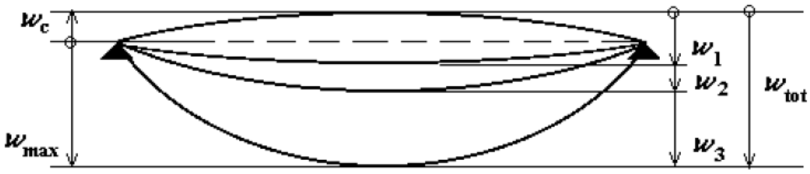

Figure 1. Definition of vertical deflections.

The definition of vertical deflections is shown in Figure 1, where

- w_c is the precamber in the unloaded structural member;
- w_1 is the initial part of the deflection under permanent loads of the relevant combination of actions;
- w_2 is the long-term part of the deflection under permanent loads;
- w_3 is the additional part of the deflection due to the variable actions of the relevant combination of actions;
- w_{tot} is the total deflection as sum of w_1, w_2, w_3;
- w_{max} is the remaining total deflection taking into account the precamber.

If the functioning or damage of the structure or to finishes, or non-structural members (e.g. partition walls, claddings) is being considered, the verification for deflection should take account of those effects of permanent and variable actions that occur after execution of the member or finish concerned, see EN 1990:2002 A1.4.3 (3) (CEN 2002).

$$\sum_{j\geq 1} G_{k,j} + Q_{k,1} + \sum_{i\triangleright 1} \psi_{0,i} Q_{k,i} \qquad (1)$$

where $G_{k,j}$ denotes the characteristic value of the jth permanent action, $Q_{k,1}$ is the characteristic value of the leading variable action, $Q_{k,i}$ is the characteristic value of the ith variable action and $\psi_{0,i}$ is the factor for combination value of a ith variable action.
If the appearance of the structure is being considered, the quasi-permanent combination should be used, see EN1990:2002 A1.4.3 (4),

$$\sum_{j\geq 1} G_{k,j} + \sum_{i\triangleright 1} \psi_{2,i} Q_{k,i} \qquad (2)$$

where $\psi_{2,i}$ is the factor for quasi-permanent value of the ith variable action.
In case of steel structures EN 1993-1-1:2005 (CEN 2005) states that the limits for vertical deflections according to Figure 1 should be specified for each project and agreed with the client and notes that the National Annexes may specify these limits.

2.2 *National Annexes*

The National Annexes (NA) of EN 1993 may specify the limits of deflections. The values given in the NAs are only suggested values, there are not obligatory rules given.
These prescriptions are quite varying and can be different depending on the function (e.g. accessible/non-accessible roof, floor etc.), the importance (main girder, purlin), the type of the carried material (plaster, brittle finish, non-brittle finish) or other conditions of the investigated element.

The deflection limit values for a general steel beam not carrying brittle finish or having some special requirement are summarized in Table 1. These values are valid for the characteristic combination of actions.

Table 1. Deflection limits for a general floor beam not carrying brittle finish

	w_{max}	w_3
UK	-	L/200
Denmark	-	L/400
Finland	L/400	-
Greece	L/250	L/300
Spain	-	L/300
Hungary	L/250	L/300

2.3 *Load representation*

In the present study maximum deflection is calculated from the characteristic load combination given in Equation 1. Considering a design situation where the leading variable action is the floor live load and the second variable action is the wind load this formula simplifies to:

$$q_n = G_k + Q_k + \psi_{0,W} W_k \tag{3}$$

where q_n represents the nominal load, G_k is the characteristic value of the permanent action, Q_k is the characteristic value of the live load, W_k is the characteristic value of the wind load and $\psi_{0,\mathrm{w}}$ is its combination factor.
To investigate the effect of the variable actions on the reliability, the following expressions can be implemented according to Gulvanessian & Holicky (2002):

$$\chi = \frac{Q_k + W_k}{G_k + Q_k + W_k} \tag{4}$$

$$k = \frac{W_k}{Q_k} \tag{5}$$

where χ represents the ratio of the variable loads to the total load (load factor) and k denotes the ratio of the wind action to the live load (variable load factor).
Using the above equations the characteristic values can be expressed as:

$$G_k = \frac{q_n}{1 + \dfrac{1 + k\chi\psi_{0,W}}{(1+k)(1-\chi)}} \tag{6}$$

$$Q_k = \frac{\chi G_k}{(1+k)(1-\chi)} \tag{7}$$

$$W_k = kQ_k \tag{8}$$

These representations of loads will be used in the following.

2.4 *Target reliabilities*

The target reliability for irreversible serviceability limit states in EN 1990 Annex C is set to be 1.5 for a 50 years reference period and 2.9 for a 1 year reference period for reliability class 2 (RC2) structural members. Class RC2 can be associated with consequence class 2, which covers structures with medium consequence for loss of human life, economic, social or environmental consequences considerable, e.g. residential and office buildings, public buildings where consequences of failure are medium.

3 RELIABILITY ANALYSIS

3.1 *Serviceability limit state*

The serviceability limit state defines what constitutes a serviceability failure. In the present paper serviceability non-compliance is deemed to occur when a deflection exceeds an allowable deflection limit as a result of flexure.
Since the deflection model of the structural steel member is based on elastic behavior, the maximum deflection of a simply supported steel beam can be expressed as:

$$\delta_{\max} = \frac{5}{384}\frac{qL^4}{EI} \tag{9}$$

where δ is the midspan deflection of the beam, q is the uniformly distributed load, L is the span, E is the modulus of elasticity and I is the second moment of inertia.
If the beam is well designed the stiffness of the section is determined in such a way, that the calculated deflection should be equal to the allowable deflection limit (if the serviceability limit state is critical). Thus the nominal value of the deflection is equal to deflection limit given by the code (subscript n means nominal values):

$$\delta_{limit} = \delta_n = \frac{5}{384}\frac{q_n L_n{}^4}{E_n I_n} \tag{10}$$

The limit state function for a member satisfying the deflection limit can be expressed as:

$$g(X) = 1 - \frac{\delta_{\max}}{\delta_{limit}} = 1 - \frac{\dfrac{5}{384}\dfrac{qL^4}{EI}}{\dfrac{5}{384}\dfrac{q_n L_n{}^4}{E_n I_n}} = 1 - \frac{qE_n I_n}{q_n EI} \tag{11}$$

In the above equation the nominal values are deterministic values given by codes and standards, however the actual values can be represented as stochastic random variables. Therefore there is a certain probability of serviceability non-compliance, which is – according to the above equation – theoretically independent from the deflection limit, assuming that the deflection limit itself is a deterministic variable.
To consider the effect of model uncertainties the limit state function for the reliability analysis should be written in the following form:

$$g(X) = 1 - \frac{\theta_E \delta_{\max}}{\theta_R \delta_{limit}} \tag{12}$$

where θ_E is the coefficient expressing the uncertainty of the action effect and θ_R is the uncertainty of the resistance model.

3.2 *Model of basic variables*

The probabilistic models of basic variables for time invariant analysis are given in Table 2.

Table 2. The simplified probabilistic models of basic variables for time invariant analysis

Description	X	Distribution	μ_X	COV(X)
Young's modulus	E	Normal	E_n	0.04
Moment of Inertia	I	Normal	I_n	0.03
Dead load	G	Normal	G_k	0.10
Live load – 50 years	Q	Gumbel	$0.6Q_k$	0.35
Live load – 5 years	Q	Gumbel	$0.2Q_k$	1.10
Wind load – 50 years	W	Gumbel	$0.7W_k$	0.25
Wind load – 1 year	W	Gumbel	$0.5W_k$	0.40
Action effect	θ_E	Lognormal	1	0.10
Resistance factor	θ_R	Lognormal	1	0.05

Column X contains the random variables and μ_X the mean value of the random variable X. The parameters have been taken from JCSS Probabilistic Model Code (JCSS 2001). Note that there are no factors given taking into account the model uncertainties in case of deflection, therefore θ_E for moments in frames and θ_R for bending moment capacity have been considered.
The applied load model was the same as the model used by Gulvanessian & Holicky (2002) which can be used for general purposes, however when the reliability of different type of structural members under particular conditions is assessed, the proposed models may be modified. The load combinations are modeled using Turkstra's combination rule to model load combinations, which states that the maxi-mum value of sum of two independent random processes occurs when one of the processes has its maximum value. Using this rule the variable load in our case is given as:

$$P_{\max} = \max \begin{Bmatrix} Q_{\max} + W^{apt} \\ Q^{apt} + W_{\max} \end{Bmatrix} \tag{13}$$

where P_{max} denotes the maximum value of the variable actions, Q_{max}, Q^{apt} and W_{max}, W^{apt} are the 50-years maximum and the arbitrary-point-in-time value of the live load and the wind load respectively.

4 RESULTS

4.1 *Remaining total deflection, w_{max}*

A parametric study has been carried out investigating the effect of changing ratio of the variable loads to total load, χ and the ratio of the wind action to the live load, k. The computations have been made using Second Order Reliability Method (SORM) with the structural reliability software COMREL 8.10 (RCP 2008). The combination factor of the second variable load (wind action) $\psi_{0,W}$ was set to 0.6 according to EN 1990 Annex A1.2.2. Figure 2 and 3 present the reliability index β and probability of serviceability non-compliance P_f versus load ratio χ respectively. The moment of inertia has been calculated from the criterion $w_{max} \leq L/250$ from the characteristic combination. It should be noted that the value of the deflection limit does not influence the serviceability non-compliance.

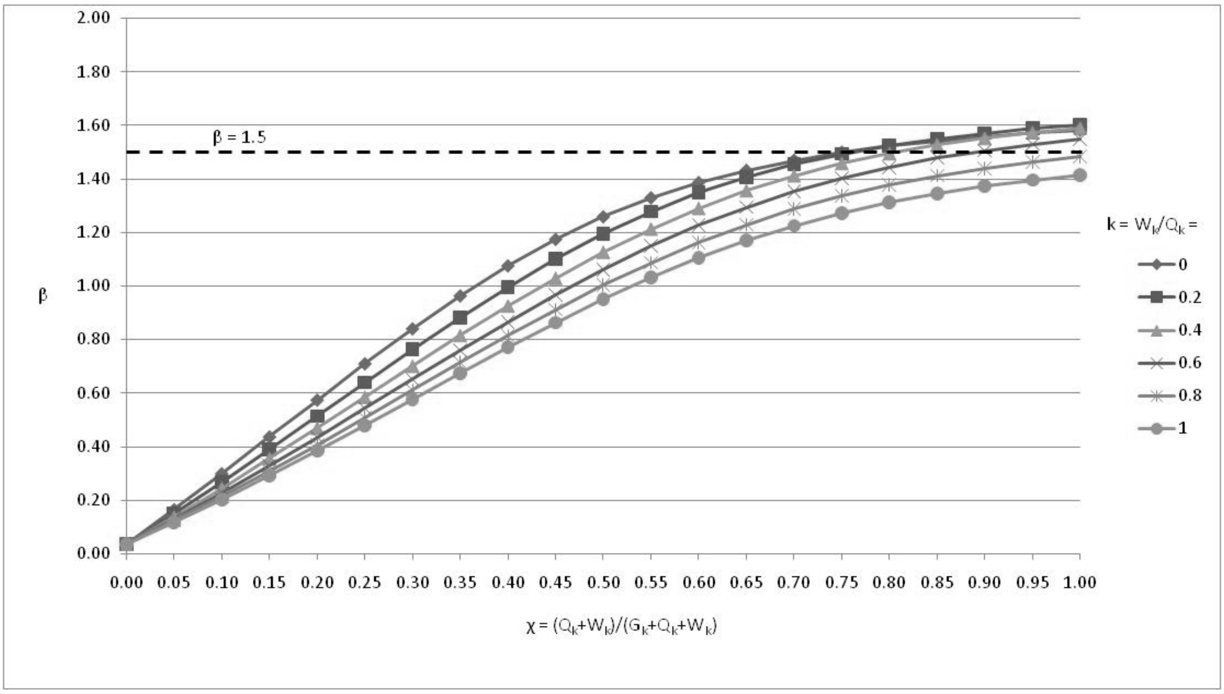

Figure 2. Reliability index β versus load ratio χ ($w_{max} \leq L/250$ – characteristic combination).

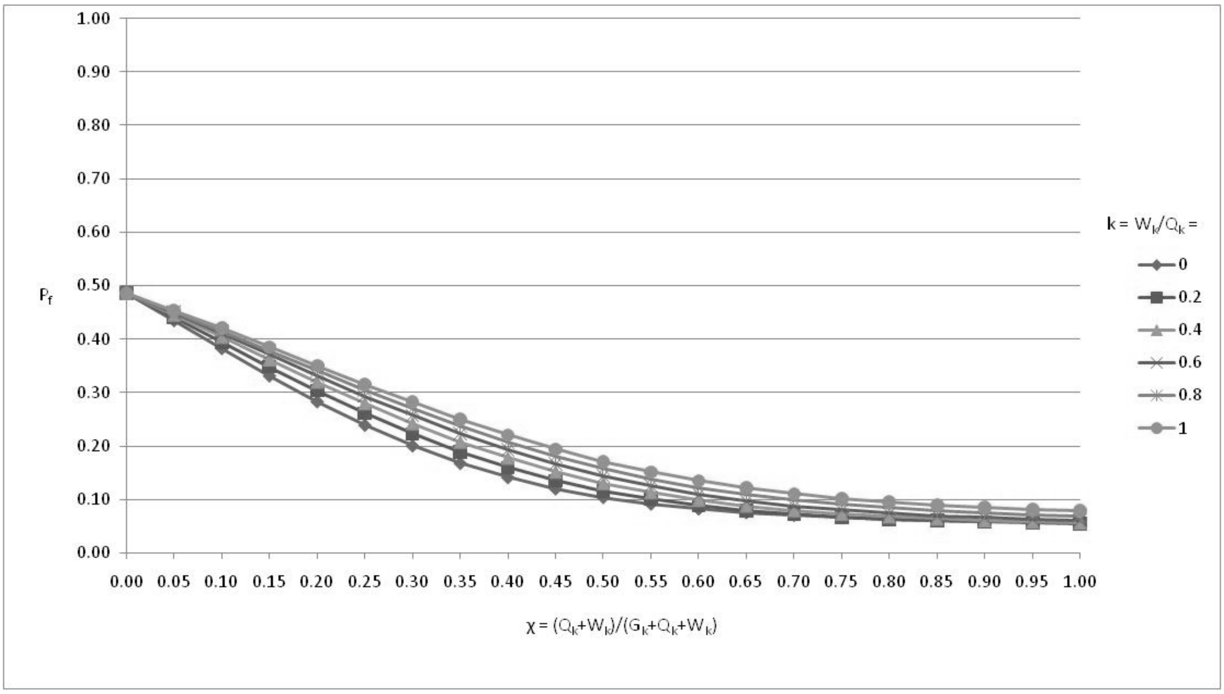

Figure 3. Probability of failure P_f versus load ratio χ ($w_{max} \leq L/250$ – characteristic combination).

From the figures it can be seen that reliability is not consistent. In case of relatively low variable loads (or high value of self-weight) the reliability is very low and therefore the probability of serviceability non-compliance is high. For the first sight it can be surprising that when the beam is loaded only by permanent actions – and designed for that situation only – the reliability index is almost equal to zero – which means that the probability of serviceability non-compliance is 50%. However it can be theoretically proven that if the deflection depends only on G, E and I – and all of them are normal random variables – β will be equal to zero. The obtained values here are slightly higher, since action effect and resistance factor have been taken into account. It should be mentioned, that from a practical point of view this case is not important, since structures are designed to carry loads. The target reliability β=1.5 given in EN 1993-1-1 Annex C is indicated in Figure 2. In most of the cases the reliability is below that value, however the actual values of β are not of great importance, since the applied model contains some simplifications. A more important issue is the remarkable differences of β for low and high values of the load factor χ. It can also be observed that increasing the second variable action (in this case the wind) the reliability decreases and so the probability of non-compliance increases, however this effect is not so significant.

4.2 *Additional part of the deflections, w_3*

In case of incremental deflection of the beam the load factor χ (the variable loads to total load) obviously has no effect on the reliability, since it is calculated only from the variable actions. Figure 4 shows the effect of varying the variable load factor k (the ratio of the wind action to the live load) on the reliability index β. The required moment of inertia in this case has been calculated from the criterion $w_3 \leq L/300$ from the characteristic combination of the variable loads. The target reliability index β=1.5 given in EC3 Annex C is indicated in this figure too. It can be seen that the values are, in most of the cases, over the target reliability – they fall below the line only for significant second variable load. For the incremental deflection criteria the reliability can be considered as consistent (standard deviation of the results is 0.07).

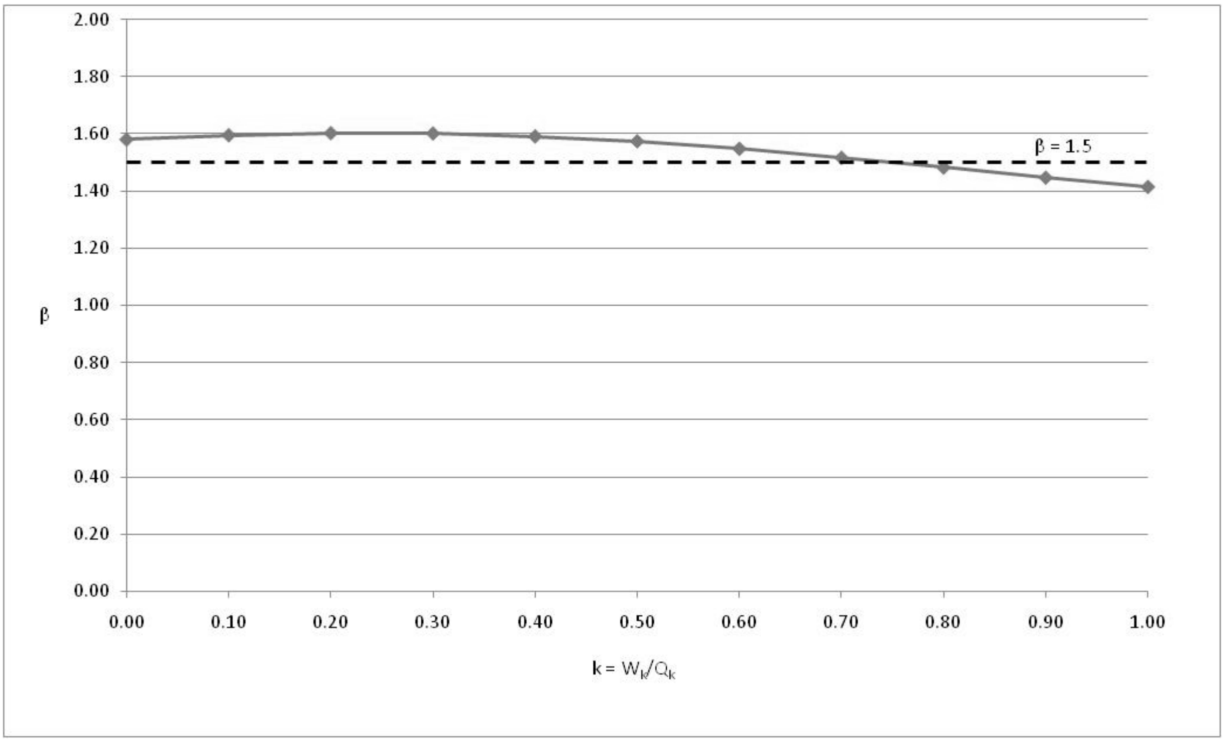

Figure 4. Reliability index β versus variable load ratio k ($w_3 \leq L/300$ – characteristic combination).

4.3 *Comparison with ULS*

In Figure 5 the different behavior of β is illustrated – in ultimate and serviceability limit state – when the load ratio is changing (in both cases k=0.5). In case of ULS we obtain higher reliability values in the low variable load region (χ<0.5) and less reliability when the variable loads are higher (χ>0.5). This is contradictory to the results of SLS, which suggests that serviceability problems occur more likely to structures where the self-weight is more relevant.

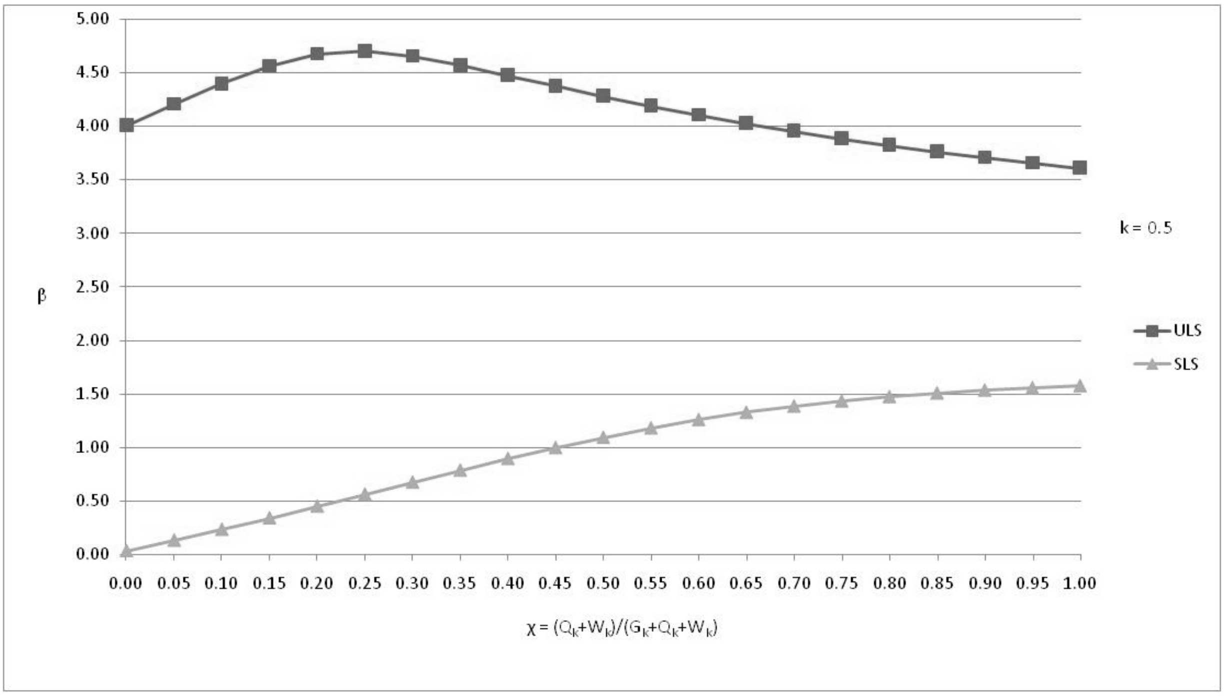

Figure 5. Comparison of β with varying load factor in case of ULS and SLS.

5 CONCLUSIONS

A second order reliability analysis has been carried out to investigate the consistency of the probability of serviceability non-compliance according to Eurocode in case of a simply supported steel beam subjected to uniform loading.
The reliability related to the total deflection has been found to be not consistent in the serviceability limit state. In case of low variable loads the reliability seemed to be low and was generally below the target value (β=1.5) given by the code. By increasing the second variable action the reliability decreased.
The reliability in case of the incremental deflection was close to the target reliability and it was relatively consistent.
Although the actual value of the deflection limit does not influence the reliability of the serviceability limit state, it is important, since this limit will decide the relation to the ultimate limit state and therefore influence the design. Unfortunately the origin of these values is not clear and they are not harmonized among the different countries, therefore an intensive investigation on this topic is essential.

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